PARAMETRIC MODELLING OF COMPLEX PERMITTIVITY AND PERMEABILITY

Tom Van den Broeck, Geert Dhondt°, Alain Barel and Luc Martens°

Vrije Universiteit Brussel, Department ELEC
Pleinlaan 2, B-1050 Brussel, Belgium
Tel. (32) (2) 629 28 86; Fax (32) (2) 629 28 50
E-mail: tvdbroec@vub.ac.be
WWW: http://homepages.vub.ac.be/~tvdbroec/

°Universiteit Gent, Vakgroep Informatietechnologie
St.-Pietersnieuwstraat 41, B-9000 Gent, Belgium
Tel. (32) (9) 264 33 33; Fax (32) (9) 264 35 93
E-mail: martens@intec.rug.ac.be
WWW: http://www.rug.ac.be/index.html

Abstract — **This paper proposes a new method for measuring the complex permittivity and permeability of materials at microwave frequencies. Starting from reflection/transmission measurements, a parametric model of ε and μ is determined using a maximum likelihood estimator.**

I. INTRODUCTION

The accurate electrical and magnetic characterization of materials at microwave frequencies is an important issue in different applications. Examples can be found in the medical (tissue classification), military (radar reflections on materials) and commercial sector (electronic materials characterization). Indeed, since the frequency used in on-chip components has steadily increased, the interconnect system has become the limiting factor in final system speed. The performance of these interconnects depends on the properties of the materials being used. Examples are the cross-talk between two lines and timing problems in digital systems due to varying delay and dispersion of the substrate being used. Therefore, it is clear that the material properties of the interconnect should be known accurately for design purposes. However, in most cases only the value at one frequency given by the manufacturer is used. Since the behaviour of the materials is frequency dependent, an alternative is to measure the properties with a nonparametric approach. This technique has the disadvantage of using large data sets, and also numerical problems make the determination of the model unreliable in some situations. Therefore, in this paper a new parametric modelling technique is proposed for both the electrical and magnetic properties of materials. This method can be used for different measurement setups ([1]-[3]). Using both reflection and transmission measurements and the uncertainty on them, a maximum likelihood estimator makes optimal use of these data. The resulting model (as a function of frequency) is very compact (typically 5 parameters) and very well suited to be used in circuit simulators.

II. FREQUENCY RESPONSE OF MATERIALS

A. Dielectric properties

The relative permittivity of a material is given by

$$\varepsilon_r = 1 + \chi \tag{1}$$

Since the susceptibility χ has a physical interpretation (relation between electrical field **E** and polarization **P**), restrictions apply to the possible functions which represent it. For example, the real and imaginary part of χ are related by the Kramers-Kroenig relations [5].

B. Magnetic properties

The magnetic properties are given by the relative permeability

$$\mu_r = 1 + \chi_m \tag{2}$$

Again, since the magnetic susceptibility χ_m relates two physical quantities (magnetization **M** and magnetic field **H**), restrictions apply to the possible functions which represent it. The system should be stable, realizable and the Kramers-Kroenig relations should be fulfilled.

III. PARAMETRIC MODEL

Considering the fact that both the permittivity and permeability of materials have a physical interpretation, several models as a function of frequency have been proposed in literature ([4], [5]). In general, all these models can be approximated in the band of interest by a rational function in the Laplace variable s:

$$\chi = \frac{\sum_{i=1}^{n} \alpha_i s^i}{\sum_{i=1}^{d} \beta_i s^i} \quad \text{and} \quad \chi_m = \frac{\sum_{i=1}^{p} \gamma_i s^i}{\sum_{i=1}^{q} \delta_i s^i} \tag{3}$$

One of the coefficients in numerator or denominator is chosen to be fixed (e.g. $\beta_0=1$) to remove the redundancy in the parameters. The advantage of this model over the nonparametric models generally used ([1]-[3]) is that we now can impose some physical constraints. Indeed, χ and χ_m should represent realizable and stable systems. Moreover, the examples will illustrate that all the poles and zeros of χ are on the negative real axis and are alternating. This means that χ can be represented by an equivalent electrical RC-network.

IV. EXPERIMENTAL SETUP

The setups we have used for estimating the parameters of our models for the permittivity and permeability are all based on transmission/reflection measurements. Two examples are shown below.

A. Waveguide setup

In this configuration a small sample is inserted inside a waveguide. S-parameters are measured with a network analyzer after calibration with a TRL method.

The theoretical model for this setup gives the following s-parameters

$$S_{11} = R_1^2 \frac{\Gamma(1-z^2)}{1-\Gamma^2 z^2}$$
$$S_{22} = R_2^2 \frac{\Gamma(1-z^2)}{1-\Gamma^2 z^2} \tag{4}$$
$$S_{21} = R_1 R_2 \frac{z(1-\Gamma^2)}{1-\Gamma^2 z^2}$$

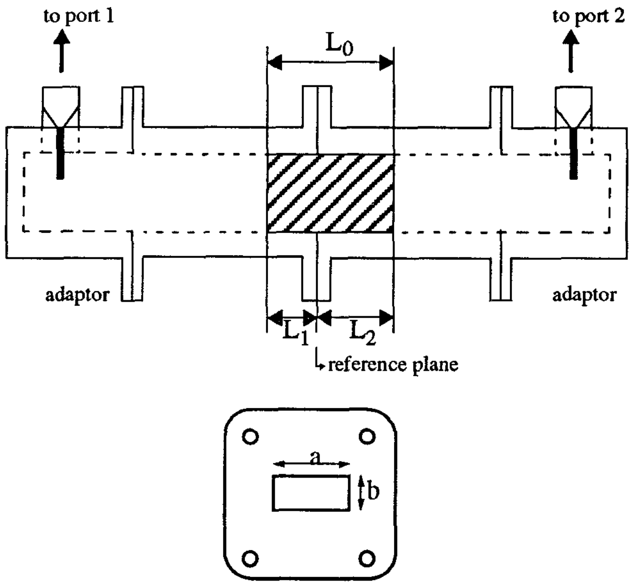

Fig. 1. Waveguide configuration (dimensions WR-90 waveguide: a=22.86 mm, b=10.16 mm)

with R_1 and R_2 transmission through air and z through the material

$$R_1 = e^{\gamma_0 L_1}$$
$$R_2 = e^{\gamma_0 L_2} \tag{5}$$
$$z = e^{-\gamma L_0}$$

and where the reflection factor Γ at the interface is defined by

$$\Gamma = \frac{\frac{\gamma_0}{\mu_0} - \frac{\gamma}{\mu}}{\frac{\gamma_0}{\mu_0} + \frac{\gamma}{\mu}} \tag{6}$$

The transmission factor T at the interface is given by

$$T^2 = 1 - \Gamma^2 \tag{7}$$

The propagation constant γ in the case of a TE_{10} mode in waveguide is given by

$$\gamma = \sqrt{\frac{\pi^2}{a^2} - \omega^2 \mu \varepsilon} \tag{8}$$

with μ and ε the permeability and permittivity respectively and a the waveguide width.

B. Free space setup

Using the configuration of Fig. 2., both S_{11} and S_{21} can be measured. For this experiment, we used only S_{21} because S_{11} is more critical to calibration errors. L_1 and L_2 are the distances from the input reference plane and output reference plane to the sample respectively, while L_0 is the sample thickness.

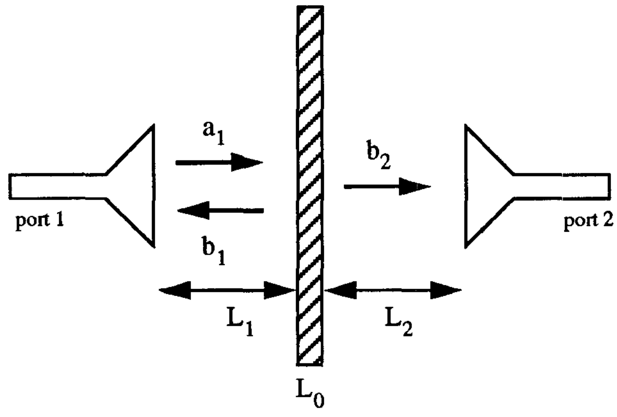

Fig. 2. Free space S-parameter setup

The same equations hold as for the waveguide setup, where the propagation constant γ now becomes

$$\gamma = \omega\sqrt{\varepsilon\mu} \tag{9}$$

Special care has to be taken that the waves at the sample can be considered to be plane and that normal incidence is used.

V. MAXIMUMUM LIKELIHOOD ESTIMATOR

The estimation of the parameters is carried out using a maximum likelihood approach, similar to the one described in [6]. The maximum likelihood estimator (MLE) requires the measured spectra S_{11m} and S_{21m} of the system under investigation, as well as the perturbing noise variances $\sigma_{11}{}^2$ and $\sigma_{21}{}^2$ and the covariance $cov_{11,21}$. The noise sources N_{11} and N_{21} are assumed to be zero mean complex normally distributed (it can be shown however that the MLE is robust with respect to this assumption [8])

$$\begin{aligned} S_{11m} &= S_{11m} + N_{11} \\ S_{21m} &= S_{21m} + N_{21} \end{aligned} \tag{10}$$

The spectra S_{11m} and S_{21m} are obtained by taking the complex average of the measurements. In this step also the variances are determined. The cost function in ML sense is now given by

$$C = \sum_{k=1}^{F} \begin{pmatrix} S_{11,m}(\omega_k) - S_{11}(\omega_k, P) \\ S_{21,m}(\omega_k) - S_{21}(\omega_k, P) \end{pmatrix}^H \cdot \ldots \\ C_v^{-1} \begin{pmatrix} S_{11,m}(\omega_k) - S_{11}(\omega_k, P) \\ S_{21,m}(\omega_k) - S_{21}(\omega_k, P) \end{pmatrix} \tag{11}$$

where $C_v(\omega_k)$ is the covariance matrix of the measured s-parameters, P are the model parameters and index m denotes measured data. A total of F angular frequencies ω_k are being considered. Equation (11) gives rise to a nonlinear minimization problem, which can be tackled by a Levenberg-Marquardt method to enlarge the convergence region. The parameters which we will determine are the sample thickness L_0, the input and output connecting lengths L_1 and L_2, the coefficients for the rational models of χ and χ_m and for the waveguide experiment the width a. For L_0, L_1, L_2 and a, good starting values are available. For χ and χ_m, we start with a low order model (e.g. 0/1). The starting values are obtained from the approximate mean values of the permittivity and permeability over the band of interest. The order selection n/d (n for numerator and d for denominator) is tackled by gradually increasing the order and looking at the cost function (11). If no substantial decrease of the cost function is obtained anymore, the model order is fixed. In [6] it is shown that if no model errors are present, the theoretical value of the cost function equals

$$C = C_{noise} = 2F - \frac{n_p}{2} \tag{12}$$

where F is the number of frequencies considered and n_p is the number of parameters. However, in our case model errors will be present (e.g. due to residual calibration errors or non-perfect sample geometry). Therefore, the cost function will be larger. The cost function due to model errors C_{model} gives an idea of the quality of the obtained model.

$$C = C_{noise} + C_{model} \tag{13}$$

The advantage of the parametric MLE method with respect to the nonparametric methods ([1]-[3]) is that we now obtain a compact model and that numerical instabilities for low loss materials [2] are avoided. Moreover, by combining transmission and reflection measurements (in contrast to [7]) and their uncertainties, optimal use is made of the information available in the measured data.

VI. EXPERIMENTAL RESULTS

Experiments were performed on a 48.3 mm long sample of bakeliet, inserted in a WR-90 waveguide. For the susceptibility, an order 2/2 model was used, while for the magnetic susceptibility an order 0/0 model was used. Both reflection and transmission measurements were taken into account, as well as their measurement noise. Amplitude errors are within 0.1 dB, phase errors within 1 degree (Fig. 3.). Results were compared with the nonparametric Nicolson-Ross method. Good agreement was found (Fig. 4.). The small differences in permittivity can be explained by the fact that Nicolson-Ross is very sensitive to sample alignment, while in the parametric method the alignment parameters are estimated.

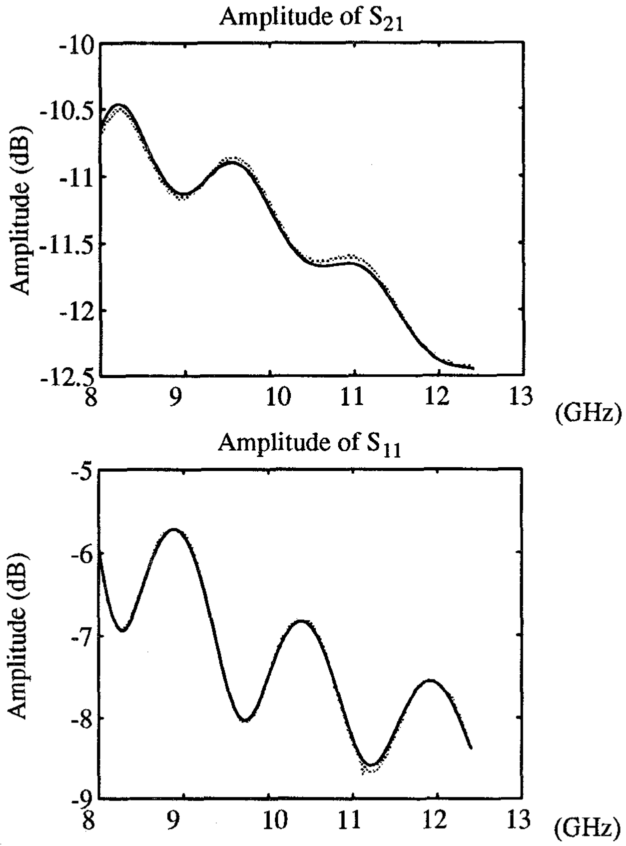

Fig. 3. Bakeliet: modelled (solid) and measured (dotted) S_{21} and S_{11}

VII. CONCLUSIONS

In this paper a new method for determining the permittivity and permeability of materials from transmission/reflection measurements based on a parametric model is proposed. Using a maximum likelihood estimator, the parameters of the model are estimated. The model errors made can be evaluated. The resulting model respects the physical constraints on permittivity and permeability, and also numerical instabilities typical for a non-parametric approach are avoided.

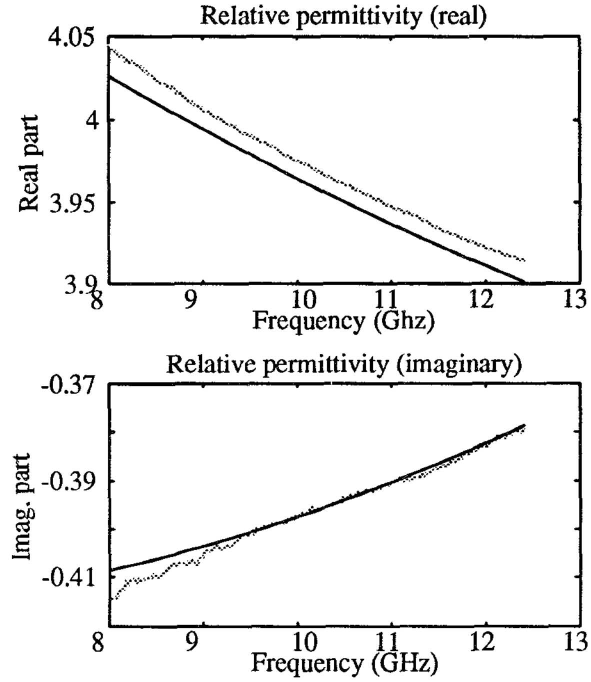

Fig. 4. Bakeliet: real and imaginary part of the relative permittivity: MLE method (solid) and Nicolson Ross method (dotted)

VIII. REFERENCES

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